A new topic course on Stochastic Differential Equations

1. Name of instructor

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2. Course information

This is a new three credit course topic course at University of Alberta. It is under an existing topic course number with a subtitle:

MATH 663 Topics in Applied Mathematics I: Stochastic differential equations.

It is planed to meet Tuesdays and Thursdays from 11:00am-12:20pm Mountain time (subject to modification). It is lectured and recorded via zoom. Everyone with an valid email address (University if Alberta, inside or outside Canada) can enrol the course via

https://eclass-cpd.srv.ualberta.ca/course/view.php?id=814

3. Abstract

This is a one semester three credit hour course and meet twice a week, tentatively Tuesdays and Thursdays from 11:00-12:20.

It is about the theory and applications of stochastic differential equations driven by Brownian motion.

The stochastic differential equations have found applications in finance, signal processing, population dynamics and many other fields. It is the basis of some other applied probability areas such as filtering theory, stochastic control and stochastic differential games. To balance the theoretical and applied aspects and to include as much audience as possible, we shall focus on the stochastic differential equations driven only by Brownian motion (white noise). We will focus on the theory and not get into specific applied area (finance, signal processing, filtering, control and so on).

We shall first briefly introduce some basic concepts and results on stochastic processes, in particular the Brownian motions. Then we will discuss stochastic integrals, Itô formula, the existence and uniqueness of stochastic differential equations, some fundamental properties of the solution. We will concern with the Markov property, Kolmogorov backward and forward equations, Feynman-Kac formula, Girsanov formula. We will also concern with the ergodic theory and other stability problems. We may also mention some results on numerical simulations, Malliavin calculus and so on.

4. Course prerequisites

Some preparation on mathematical analysis and probability theory.

5. Reference texts

The main reference book for this course is

- Øksendal, B. Stochastic differential equations. An introduction with applications. Sixth edition. Universitext. Springer-Verlag, Berlin, 2003. xxiv+360 pp. ISBN: 3-540-04758-1
- Karatzas, Ioannis; Shreve, Steven E. Brownian motion and stochastic calculus. Second edition. Graduate Texts in Mathematics, 113. Springer-Verlag, New York, 1991. xxiv+470 pp. ISBN: 0-387-97655-8
- Klebaner, Fima C. Introduction to stochastic calculus with applications. Third edition. Imperial College Press, London, 2012. xiv+438 pp. ISBN: 978-1-84816-832-9; 1-84816-832-2

Some other reference books are

- Ikeda, N.; Watanabe, S. Stochastic differential equations and diffusion processes. Second edition. North-Holland Mathematical Library, 24. North-Holland Publishing Co., Amsterdam; Kodansha, Ltd., Tokyo, 1989. xvi+555 pp. ISBN: 0-444-87378-3
- Protter, P. E. Stochastic integration and differential equations. Second edition. Version 2.1. Corrected third printing. Stochastic Modelling and Applied Probability, 21. Springer-Verlag, Berlin, 2005. xiv+419 pp. ISBN: 3-540-00313-4
- Revuz, D.; Yor, M. Continuous martingales and Brownian motion. Third edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 293. Springer-Verlag, Berlin, 1999. xiv+602 pp.
- Durrett, R. Stochastic calculus. A practical introduction. Probability and Stochastics Series. CRC Press, Boca Raton, FL, 1996. x+341 pp. ISBN: 0-8493-8071-5
- Jeanblanc, M.; Yor, M.; Chesney, M. Mathematical methods for financial markets. Springer Finance. Springer-Verlag London, Ltd., London, 2009. xxvi+732 pp. ISBN: 978-1-85233-376-8
- Hasminskii, R. Z. Stochastic stability of differential equations. Translated from the Russian by D. Louvish. Monographs and Textbooks on Mechanics of Solids and Fluids: Mechanics and Analysis, 7. Sijthoff & Noordhoff, Alphen aan den RijnGermantown, Md., 1980. xvi+344 pp. ISBN: 90-286-0100-7
- Hu, Y. Analysis on Gaussian spaces. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017. xi+470 pp. ISBN: 978-981-3142-17-6

- Kloeden, P. E.; Platen, E. Numerical solution of stochastic differential equations. Applications of Mathematics (New York), 23. Springer-Verlag, Berlin, 1992. xxxvi+632 pp. ISBN: 3-540-54062-8

6. Description of how the course will be disseminated network-wide

The course contents are chosen so that they may be applicable to wide range of areas so researchers from various fields may be interested in these materials. This course is not specifically designated to students in finance or students in electrical engineering. But to a wider audience who want to have some more knowledge on the modern state-of-art theory of stochastic differential equations.

The course will be delivered through remote means. There will be slides for each lecture. We will use the writing on the screen to discuss problems and answers so that it is as close as to the classroom presentation.

7. Grading scheme

The grade will be based on the homework problems (70%), to be selected from the book of Øksendal and Klebaner. The remaining 30% will be from a project to be determined after half of the course. Individual special needs cannot requested via email to the instructor and can be accommodated if approved.