

## PIMS NETWORK COURSE: ANALYTIC NUMBER THEORY II (GREG MARTIN, UBC)

This course is a second graduate course in number theory, intended to follow Analytic Number Theory I taught by Prof. Habiba Kadiri (University of Lethbridge) in Fall 2022. This course will be taught in Winter 2023, every Monday, Wednesday, and Friday from 10:00–10:50am Pacific time. This course also precedes the summer school “Inclusive Paths in Explicit Number Theory” in Summer 2023 and is designed to give students the ideal preparation for that summer school program. All three of these events are part of the current PIMS CRG “ $L$ -functions in Analytic Number Theory”.

Depending upon the availability of dedicated hybrid lecture rooms in the PIMS site at UBC, the lectures will either be held in one of those rooms, or else by Zoom for all participants, ensuring either way that the learning experience will be robust and equitable for students both at UBC and at other PIMS sites. The applicant has experience running PIMS Network Courses in both formats, having successfully taught both “Distribution of prime numbers and zeros of Dirichlet  $L$ -functions” (Winter 2011) and “Comparative prime number theory” (Winter 2021).

We will start by learning about Dirichlet characters and sums involving them (character sums over intervals and Gauss sums), which is sufficient preparation to prove Dirichlet’s theorem on the infinitude of primes in arithmetic progressions. We will then study Dirichlet  $L$ -functions and their zeros, which will allow us to prove the stronger prime number theorem in arithmetic progressions, including the explicit formula for the error term. From this result we can continue into limiting distributions of error terms in analytic number theory and describe the foundations of comparative prime number theory, which includes “prime number races” as well as related problems of Mertens, Polyá, and Turán. We will assume, but quickly review as we go, results about arithmetic functions and global prime counting functions, as well as the complex-analytic proof of the prime number theorem itself. As part of the PIMS CRG on  $L$ -functions in Analytic Number Theory, the course will include up to eight guest lectures by experts in the field (tentatively including Leo Goldmakher for character sums, Lucile Devin for limiting distributions, Daniel Fiorilli for prime number races, and Nathan Ng for the Mertens sum).

The evaluation for this course will consist of 1–2 write-ups (5–10 pages) of specific topics or results related to the subject matter, which will be completed either individually or in teams depending on the enrollment in the course. Students outside UBC will receive credit for this course according to the Western Deans Agreement.

The main textbook for this course will be the book by H. L. Montgomery and R. C. Vaughan, *Multiplicative Number Theory I: Classical Theory* (Cambridge University Press, 2006). Other effective resources include:

- H. Davenport, *Multiplicative Number Theory*
- T. M. Apostol, *Introduction to Analytic Number Theory*
- H. Iwaniec and E. Kowalski, *Analytic Number Theory*
- P. T. Bateman and H. G. Diamond, *Analytic Number Theory: An introductory course*