Math 535: Real Groups (Lie Theory II)
Spring Term, 2023

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Course Website	`http://www.math.ubc.ca/~lior/teaching/2223/535_W23/`
Contact me at	`lior@math.ubc.ca` or at MATX 1112
My Website	`http://www.math.ubc.ca/~lior/`
Class	TBA + Zoom
Office Hours	TBA + Zoom
Textbook	None required; see below for recommendations
(Informal) Prerequisites	Group theory, Lie algebras, point-set topology, functional analysis

About the course

This will be a graduate course on the structure and representation theory of real Lie groups. The course will have four parts:

1. Introduction: topological groups, representation theory, and the Peter–Weyl Theorem.
2. Lie groups: basic definitions and differential geometry.
3. Compact Lie groups: maximal tori, roots and weights, representation theory, Weyl character and integration formulae.
4. Real Lie groups: Lie algebras, Poincare–Birkhoff–Witt, Cartan subalgebras, roots and weights, symmetric spaces, introduction to infinite-dimensional representations.

As a graduate course there are no strict pre-requisites, but the course will be easier to follow for students with a general graduate background including real analysis and point-set topology, functional analysis, some algebra, and undergraduate group theory and linear algebra.

- Specific background: while we will develop the notion of a Lie algebra from scratch as well as the theory of roots, weights, etc, the course will be easier going for students who have previously seen Lie algebras and the classification of complex (semi)simple Lie algebras, for example by taking UBC MATH 534.

Textbooks

The books [2, 4] give general introductions to Lie groups and are the main references for parts (1),(2),(4). The book [1] is a complete reference to the central part of the course – the compact case. The early sections of [3] review the structure theory of real semisimple groups and the representation theory of compact groups, but the main focus is on infinite-dimensional representations.

Evaluation and grading; levels of participation

The grade will be based on up to ten problem sets (*there will be no final exam*). Students should note that each problem set is *comprehensive* and will contain many problems: more than what a reasonable student might do, so each participant will need to manage their workload.

Remote participation

The course will be simultaneously broadcast on Zoom and open for remote participation, including as a PIMS network-wide course.

1. Students at Canadian PIMS member universities may apply for graduate credit via the Western Deans' Agreement. Please be advised, in some cases students must enroll 6 weeks in advance of the term start date and will typically be required to pay ancillary fees to the host institution (as much as $270) or explicitly request exemptions. Please follow the link above for details of fees at specific sites.

2. Students at universities not covered by the WDA but which are part of the Canadian Association for Graduate Studies may still be eligible to register for this course under the Canadian University Graduate Transfer Agreement. Details of this program vary by university and are also typically subject to ancillary fees. Please contact your local Graduate Student Advisor for more information.

3. Anyone (anywhere in the world) who is interested in mathematics and would like to learn some Lie Theory is welcome to attend the lectures as well as the online office hours. To be clear I will only mark the homework of registered participants.

All remote and in-person participants are strongly encourage to **interrupt the lectures** repeatedly with their questions (even "can you explain that again?" or "what do you mean by 'continuous'?"). Such questions are welcome, encouraged, and are **absolutely essential** for learning to take place in a course of this type.

References

[1] Bröcker–tom Dieck, *Representations of Compact Lie Groups*

[2] Knapp, *Lie Groups Beyond an Introduction*

[3] Knapp, *Representation Theory of Semisimple Groups: An Overview Based on Examples*

[4] Warner, *Foundations of Differentiable Manifolds and Lie Groups*