Proposal of a network topic course on "Stochastic Differential Equations"

Fall 2022

1. Name of instructor

Yaozhong Hu, Department of Mathematical and Statistical Sciences, University of Alberta at Edmonton.

Email: yaozhong@ualberta.ca

Tel: 1 780 809 7003

2. Title of course

Stochastic differential equations. (It will be under the university's existing topic course MATH 663 Topics in Applied Mathematics I)

3. Abstract

This is a one semester three credit hour course and it is planned to meet Tuesdays and Thursdays from 11:00am-12:20pm Mountain time for Fall 2022. It is lectured via zoom.

It is about the theory and applications of stochastic differential equations driven by Brownian motion.

A stochastic differential equation (SDE) is a differential equation in which the rate of change is determined by the state of the system itself, some external known forces and some unknown external forces as well. The noise in the system is given by random white noise calculated as the derivative of Brownian motion or the Wiener process. However, other types of random behaviour are possible, such as jump processes. Random differential equations are conjugate to stochastic differential equations. This course will concentrate on stochastic differential equations driven by Brownian motions.

The stochastic differential equations are used to model various phenomena such as unstable stock prices or physical systems subject to thermal fluctuations. They have found applications in finance, signal processing, population dynamics and many other fields. It is the basis of some other applied probability areas such as filtering theory, stochastic control and stochastic differential games. To balance the theoretical and applied aspects and to include as much audience as possible, we shall focus on the stochastic differential equations driven only by Brownian motion (white noise). We will focus on the theory and not get into specific applied area (finance, signal processing, filtering, control and so on).

We shall first briefly introduce some basic concepts and results on stochastic processes, in particular the Brownian motions. Then we will discuss stochastic integrals, Itô formula, the existence and uniqueness of stochastic differential equations, some fundamental properties of the solution. We will concern with the Markov property, Kolmogorov backward and forward equations, Feynman-Kac formula, Girsanov formula. We will also concern with the ergodic theory and other stability

problems. We may also mention some results on numerical simulations, Malliavin calculus and so on.

4. Course prerequisites

Some preparation on mathematical analysis and probability theory.

5. Reference texts

The main reference book for this course is

- Øksendal, B. Stochastic differential equations. An introduction with applications. Sixth edition. Universitext. Springer-Verlag, Berlin, 2003. xxiv+360 pp. ISBN: 3-540-04758-1
- Karatzas, Ioannis; Shreve, Steven E. Brownian motion and stochastic calculus. Second edition. Graduate Texts in Mathematics, 113. Springer-Verlag, New York, 1991. xxiv+470 pp. ISBN: 0-387-97655-8
- Klebaner, Fima C. Introduction to stochastic calculus with applications. Third edition. Imperial College Press, London, 2012. xiv+438 pp. ISBN: 978-1-84816-832-9; 1-84816-832-2

Some other reference books are

- Ikeda, N.; Watanabe, S. Stochastic differential equations and diffusion processes. Second edition. North-Holland Mathematical Library, 24. North-Holland Publishing Co., Amsterdam; Kodansha, Ltd., Tokyo, 1989. xvi+555 pp. ISBN: 0-444-87378-3
- Protter, P. E. Stochastic integration and differential equations. Second edition. Version 2.1. Corrected third printing. Stochastic Modelling and Applied Probability, 21. Springer-Verlag, Berlin, 2005. xiv+419 pp. ISBN: 3-540-00313-4
- Revuz, D.; Yor, M. Continuous martingales and Brownian motion. Third edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 293. Springer-Verlag, Berlin, 1999. xiv+602 pp.
- Durrett, R. Stochastic calculus. A practical introduction. Probability and Stochastics Series. CRC Press, Boca Raton, FL, 1996. x+341 pp. ISBN: 0-8493-8071-5
- Jeanblanc, M.; Yor, M.; Chesney, M. Mathematical methods for financial markets. Springer Finance. Springer-Verlag London, Ltd., London, 2009. xxvi+732 pp. ISBN: 978-1-85233-376-8
- Hasminskii, R. Z. Stochastic stability of differential equations. Translated from the Russian by D. Louvish. Monographs and Textbooks on Mechanics of Solids and Fluids: Mechanics and Analysis, 7. Sijthoff & Noordhoff, Alphen aan den RijnGermantown, Md., 1980. xvi+344 pp. ISBN: 90-286-0100-7
- Hu, Y. Analysis on Gaussian spaces. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017. xi+470 pp. ISBN: 978-981-3142-17-6

- Kloeden, P. E.; Platen, E. Numerical solution of stochastic differential equations. Applications of Mathematics (New York), 23. Springer-Verlag, Berlin, 1992. xxxvi+632 pp. ISBN: 3-540-54062-8

6. Description of how the course will be disseminated network-wide

The course contents are chosen so that they may be applicable to wide range of areas so researchers from various fields may be interested in these materials. This course is not specifically designated to students in finance or students in electrical engineering. But to a wider audience who want to have some more knowledge on the modern state-of-art theory of stochastic differential equations.

The course will be delivered through remote means. There will be slides for each lecture. We will use the writing on the screen to discuss problems and answers so that it is as close as to the classroom presentation.

7. Grading scheme

The grade will based on the homework problems (70%), to be selected mainly from the book of Øksendal and Klebaner. The remaining 30% will be from a project to be determined after half of the course.

8. Expected student demand across the PIMS network for such a course

The instructor taught this course via zoom Winter 2021. There were a relatively big enrolment (about 19 students). One reason that the instructor plans to teach this course the coming fall semester is because one student from University of Ottawa and some students here from University of Alberta expressed their interest in taking this course. In general, it is expected that some students from probability, statistics, mathematical finance, dynamical systems, electrical engineering and so on may be interested in this course. The instructor hopes that it will have a similar enrolment (18 students). However, the precise enrolment cannot be predicted at this moment.

9. Instructor experience with disseminating courses remotely and description of any possible challenges anticipated with the network-wide dissemination

The instructor has given remote courses on "elementary analysis and MATH 663 Topics in Applied Mathematics I: Stochastic differential equations (This proposed course). The instructor has also given three short courses and delivered numerous talks remotely. He has gained an abundant experience on delivering course lectures. The pdf file of the lecture slides will be used during the lectures. The ipad and the associated teaching and note taking softwares (such as notability) will be used to enhance the effectiveness of the lectures. The annotation of zoom can be used to enhance the communication with students to help them to understand the material. Students are encouraged to ask questions.

The instructor plans to pay special attention to know how the material is digested by the students and if the students understand the lecture or not. Since the body of the students are mostly more advanced undergraduate students or graduate students the instructor also plans to get feedback from the students to modify the contents and paces of the lectures.

10. Budget

The use of zoom should be covered by the university of Alberta. The homework collection and grading will be through university's eclass (mainly via a software called "assign2") which is also covered by the university. If the enrolment is high (15 students or above), the instructor plans to get a postdoc in the field to grade homework.

11. Intended syllabus The contents are

1) Brownian motions.
2) Properties of BrownianMotion Paths
3) Martingale and Markov Property of Brownian Motion
4) Hitting Times and Exit Times
5) Maximum and Minimum of Brownian Motion
6) Reflection Principle and Zeros of Brownian Motion Arcsine Law
7) Definition of Itô Integral
8) Itô Formula
9) Itô Processes in Higher Dimensions
10) Definition of Stochastic Differential Equations
11) Markov Property of Solutions
12) Weak Solutions to SDEs
13) Kolmogorov Backward and Forward Equations
14) Diffusion Processes
15) Martingales and Dynkins Formula
16) Calculation of Expectations and PDEs
17) Explosion and non-explosion
18) Recurrence and Transience
19) Ergodic theory
20) Feynman-Kac formula
21) Girsanov formula.
22) Euler-Maruyama schemes

23) Milstein scheme

24) Nnonlinear filtering theory

25) Stochastic control