# MATH 800 : Computer Algebra

Instructor : Michael Monagan
Fall 2023

## Course Outline

**1 Getting Started**

- Analysis of algorithms tutorial.
- Programming in Maple tutorial.
- Polynomial interpolation algorithms.

**2 Algorithms for Linear Algebra over Integral Domains.**

- Fraction-free algorithms for computing $\det(A)$ and solving $Ax = b$.
- The Berkowitz division free algorithm for computing $\det(A - \lambda I)$.
- Solving $Ax = b$ over $\mathbb{Q}$ using $p$-adic lifting and rational number reconstruction.

**3 The Fast Fourier Transform and Applications.**

- The FFT and fast multiplication.
- Fast division and fast multi-point evaluation using the FFT.

**4 Data Structures and Algorithms for Multivariate Polynomials.**

- Data structures for multivariate polynomials.
- Multiplication and division using repeated merging.
- Multiplication and division using binary heaps.

**5 Sparse Multivariate Polynomial Interpolation.**

- Sparse polynomials. Black box representations. The Schwartz-Zippel Lemma.
- Zippel's sparse interpolation and polynomial GCD computation.
- Ben-Or Tiwari interpolation. Kronecker substitutions.

**6 Groebner Bases and Applications.**

- Ideals in polynomial rings and computing in quotient rings.
- Monomial orderings. Ideal membership and polynomial division.
- The Hilbert basis theorem. Groebner bases. Buchberger's algorithm.
- The elimination theorem. Applications. Groebner bases in Maple.

**7 Computing with Algebraic Numbers.**

- Computing in $\mathbb{Q}(\alpha)$ and $\mathbb{Q}(\alpha_1, \alpha_2, ..., \alpha_n)$.
- Primitive elements, norms and resultants.
- Factoring polynomials over $\mathbb{Q}(\alpha)$.

**Grading**
Assignments 75% (best 6 of 7 assignments, one per topic).
Course Project 25%