Formalization of Mathematics
a PIMS network course

Basic Information

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Course webpage: TBA
Lectures: TBA
Office Hours: TBA or by appointment

Abstract

The last few years have seen amazing advances in interactive proof assistants and their use in mathematics. For example, Lean's mathematics library `mathlib` now has over one million lines of code and is still growing in a significant rate. Furthermore, recent highly celebrated successes in the subject, such as the completion of the sphere eversion project and the liquid tensor experiment, suggest that we are approaching a paradigm shift in mathematics, where cutting edge research can be formally verified in a relatively short amount of time.
This course will serve as an introduction to the formalization of mathematics, using the `Lean4` interactive proof assistant and its mathematics library `Mathlib4`.

Textbook

There is no required textbook for this course. Lecture notes and/or slides will be provided throughout the term. Any code associated with this course will be hosted on `github` and available to students throughout the term. Additional (freely available) sources will be provided, as needed.

Prerequisites

There are no strict mathematical prerequisites, but a certain level of mathematical maturity will be assumed. Although not strictly required, it would be useful for students to have a *minor* level of familiarity with interactive theorem proving, for example at the level of the *natural number game*.

Outline

The following is a general outline of the topics we expect to cover. We will not spend much time discussing the mathematical concepts themselves, as some level of familiarity with these concepts is assumed. Rather, the focus will be primarily on the *formalization* of these concepts using Lean.

0. Logistics: installing Lean, tooling, editors, github, etc.
1. Basics: The Curry–Howard correspondence; structures and classes; inductive constructions; Lean's simplifier; Lean's typeclass system; more advanced tactics.
2. Order: The order hierarchy; posets, lattices, etc.; Galois connections and insertions.
3. Algebra: the algebra hierarchy; subobjects and morphisms; bundled, semibundled and unbundled structures.
4. Topology: the topological hierarchy; filters, limits and continutiy; uniform spaces; topological algebra.
5. Category theory: categories, functors, natural transformations; (co)limits; adjunctions.

Additional topics will be added, time permitting, based on the audience's interests.

Format

This will be an online course. Lectures will be delivered using zoom (or similar software), and recordings of lectures will be available on the course webpage. Discussions during and outside of lecture will take place on discord (or similar software). Open discussion (by text or voice) is *highly encouraged* during lectures.

Evaluation

Final grades will be based on successful completion of problem sets and a final project. Problem sets will be assigned, and must be completed, using Lean. The final project will involve a formalization of a mathematical result chosen by the student.