HODGE THEORY, DELIGNE COHOMOLOGY AND ALGEBRAIC CYCLES

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Abstract. These lectures are based on a mini course delivered at the USTC in Hefei, China, June 23 - July 12, 2014.

1. Comments

Electronic notes will be provided in this course, which will be delivered online. All lectures will be recorded and accessible to the students. Office hours will be arranged via Zoom.

2. Syllabus

• Hodge theory, formalism of mixed Hodge structures, algebraic cycles (classical).
• Cohomological machinery, a primer on spectral sequences, double complexes and the Grothendieck spectral sequences, hypercohomology.
• Kähler differentials and arithmetic de Rham cohomology, arithmetic Gauss-Manin connection, geometric Gauss-Manin connection, Milnor K-theory, arithmetic cycle class map, de Rham and Mumford-Griffiths invariants.
• Deligne cohomology I, real Deligne cohomology I, [Beilinson]-Deligne cohomology II, real [Beilinson]-Deligne cohomology II.
• Cycle class maps, classical case, generalized cycles I, generalized cycles II, cycle class map (generic point), an abridged version of the Bloch map, Beilinson rigidity, local to global issues, Abel-Jacobi map.
• Examples of regulators and normal functions, classical case and CY threefolds, Deligne cohomology and normal functions, K_1 for K3 surfaces, torsion indecomposables, K_2 and elliptic curves, horizontality issues of cycle induced normal functions.
• Arithmetic normal functions, Bloch-Beilinson filtration, approach via Lewis, (generalized) normal functions.
• The Beilinson-Hodge conjectures, formulation of the conjectures, integral formulations and the Milnor regulator, key example I, alternate take via a morphism of sites, key example II, Bloch-Kato theorem and its consequences.

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