

Math 8410 Spectral Methods for PDEs (Winter 2025)

Instructors: Shaun Lui (shaun.lui@umanitoba.ca) and Mikael Slevinsky (richard.slevinsky@umanitoba.ca).

Tentative Time: Tues, Thurs 3 - 4:15 (CDT)

Location: MH416 and Zoom link (Meeting ID: 675 4106 0780 Passcode: 626774)

Textbook: Course notes will be provided.

References:

1. J. Shen T. Tao and L.-L. Wang, Spectral methods. Algorithms, analysis and applications, Springer, 2011.
2. L.N. Trefethen, Spectral Methods in Matlab, SIAM, 2000.
3. L.N. Trefethen, Approximation Theory and Approximation Practice (Extended Ed.), SIAM 2020.
4. S. Olver, R. M. Slevinsky, and A. Townsend, Fast algorithms using orthogonal polynomials, *Acta Numerica*, 29: 573–699, 2020.

Grading Scheme: 4 Homeworks (17% each) and a project (32%). Your final grade will be calculated approximately as follows: A+ (95-100), A (86-94), B+ (80-85), B (72-79), C+ (65-71), C (60-64), D (50-59), F (0-49).

Late assignments will not be accepted, but please talk to the instructor if an health or other issues arise. You are permitted to use all resources (including generative AI) as long as you cite your sources. While you may discuss the homework questions with your colleagues, please write down your own solutions.

Prerequisite: Undergraduate analysis and PDEs. Some exposure to numerical analysis is desirable, but not necessary. Some of the homework questions will require computer programming (MATLAB or Julia, etc.)

Course Content:

1. Part I: Introduction to Spectral Methods (Shaun Lui)
 - (a) Trigonometric and orthogonal polynomials (truncation, interpolation and quadrature error estimates)
 - (b) Fourier spectral, spectral Galerkin and spectral tau methods

- (c) Spectral collocation for Poisson equation with Dirichlet BCs (convergence and condition number estimates)
 - (d) Neumann problems and fourth-order PDEs
 - (e) Other topics (Ultraspherical spectral methods, time-dependent PDEs)
2. Part II: Fast Algorithms for Orthogonal Polynomials (Mikael Slevinsky)
- (a) Synthesis and analysis
 - (b) Chebyshev polynomials and the fast discrete sine and cosine transforms
 - (c) Modification algorithms for orthogonal polynomials
 - (d) Fast approximation of the connection coefficients
 - (e) Multivariate orthogonal polynomials via Koornwinder's construction
 - (f) Time evolution with exponential integrators

Academic Integrity - Plagiarism and Cheating

The Department of Mathematics, the Faculty of Science and the University of Manitoba regard acts of academic dishonesty in quizzes, tests, examinations or assignments as serious offenses and may assess a variety of penalties depending on the nature of the offense. Acts of academic dishonesty include bringing unauthorized materials into a test or exam, copying from another student, plagiarism and examination personation. Students are advised to read section 7 (Academic Integrity) and section 4.2.8 (Examinations: Personations) in the “General Academic Regulations and Requirement” of the current Undergraduate Calendar. Note, in particular that cell phones and pagers are explicitly listed as unauthorized materials, and hence may not be present during tests or examinations. Penalties for violation include being assigned a grade of zero on a test or assignment, being assigned a grade of “F” in a course, compulsory withdrawal from a course or program, suspension from a course/program/faculty or even expulsion from the University. For specific details about the nature of penalties that may be assessed upon conviction of an act of academic dishonesty, students are referred to University Policy 1202 (Student Discipline Bylaw) and to the Department of Mathematics policy concerning minimum penalties for acts of academic dishonesty. The Student Discipline Bylaw is printed in its entirety in the Student Guide, and is also available on-line or through the Office of the University Secretary. Minimum penalties assessed by the Department of Mathematics for acts of academic dishonesty are available on the Department of Mathematics web-page. All Faculty members (and their teaching assistants) have been instructed to be vigilant and report incidents of academic dishonesty to the Head of the Department.