MATH 605F 201 2023W2 Topics in Applied Mathematics - TPC APPLIED MATH

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Applied Stochastic Analysis

Tues/Thurs 2-3:30, Math 202

Instructor:
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Office: Math 112

Office Hours: TBD (options: Tues 3:45-5:45; Thurs 3:45-5:45; Mon 4-6)

Description: This course will introduce the major tools in stochastic analysis from an applied mathematics perspective. Topics to be covered include Markov chains (both discrete and continuous), Gaussian processes, Ito calculus, stochastic differential equations (SDEs), numerical algorithms for solving SDEs, forward and backward Kolmogorov equations and their applications. It will pay particular attention to the connection between stochastic processes and PDEs, as well as to physical principles and applications. The class will attempt to strike a balance between rigour and heuristic arguments: it will assume that students have seen a little analysis, particularly in the context of studying PDEs, but will generally avoid measure theory. The target audience is graduate students in applied mathematics or related fields, who wish to use these tools in their research for modelling or simulation.

The course will be divided roughly into two parts: the first part will focus on stochastic processes, particularly Markov chains, and the second part will focus on stochastic differential equations and their associated PDEs.

Prerequisites: Good upper-level undergraduate or early graduate knowledge of: probability, linear algebra, PDEs, and ODEs. Some prior experience with numerical analysis is helpful but not necessary.

Homework

Homework will be a critical part of the course. Lectures will mostly be theory, and examples, extensions, and applications will be assigned as homework problems. You must do these if you want to learn something from the course.

Many homework assignments will involve some computing. We will use Pluto notebooks from the high-level computing language Julia for the examples. You can use Pluto or just regular Julia for submitted code.

There will be weekly homework assignments, generally due **Thursday at 12pm**. Late homework will be penalized 5% per day, and will be accepted for up to 7 days. The lowest 2 homework scores will be dropped.

Homework is found in the "Assignments" folder of Canvas. Links are in the table below.

HOW TO HAND IT IN -- TBD

You are *strongly encouraged to work with others*, on the homework problems and to study. However, you must *write up your own solutions*. Solutions that are identical, or nearly so, will be considered as plagiarism and will be treated accordingly. The best way to ensure this doesn't happen is, (once you have discussed the problems with others), to find a place on your own to sit and write your solutions, away from the input of others. It is also a good practice to acknowledge the students you worked with, eg at the top of your assignment.

Exam or Project

You can choose to do either a final exam or a project. More information is here: TBD.

Grading Scheme

Homework 60%
Final Exam or Project 40%

References

The lectures will follow these notes quite closely, at a pace of roughly one chapter per week:

- M. Holmes-Cerfon, *Applied Stochastic Analysis* **[pdf] (https://canvas.ubc.ca/courses/139375/files/30829520?wrap=1) (https://canvas.ubc.ca/courses/139375/files/30829520/download?download_frd=1)** .
  (This is currently being editted for publication in the Courant Lecture Notes series - hence, please do not post this pdf publically.)

Some other references that are highly recommended include:

- W. E, T. Li, E. Vanden-Eijnden. *Applied Stochastic Analysis.*
  The book that is the most similar to our course.
- G. A. Pavliotis. *Stochastic Processes and Applications*.
  This is a good references for the second part of the course, on stochastic calculus and associated

PDEs. You can access a free pdf via the UBC library.

- G. Grimmett and D. Stirzaker, *Probability and Random Processes.*
  Great reference for probability theory and Markov chains.
- C. Gardiner, *Stochastic Methods: A Handbook for the Natural and Social Sciences*.
  A fantastic reference to have on the shelf. Easy to look up formulas, lots of physical applications.

Other good references include:

- L. Koralov and Y. G. Sinai, *Theory of Probability and Random Processes* (Springer)
  More rigorous/theoretical construction of stochastic processes.
- B. Oksendal, *Stochastic Differential Equations* (Springer)
  Excellent introduction to stochastic calculus.
- I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus* (Springer)
  More rigorous/theoretical construction of Brownian motions and its various properties.
- R. Durrett, *Essentials of Stochastic Processes* (Springer)
  Highly accessible reference on Markov chains and martingales.
- R. Durrett, *Stochastic Calculus: A Practical Introduction*
  A more theoretical construction than we will see, but is a useful and accessible text.
- P. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations* (Springer)
  This is the bible on numerically solving SDEs.

Draft Schedule (updated as we go)

Week	**Topics**	**Readings & Materials**
January 8-12	probability review; intro to Markov chains	Appendix 0.1, 0.2, Chapter 2 (2.1, 2.2)
January 15-19	Markov chains (I): discrete-time Markov chains	Chapter 2
January 22-26	Markov chains (II): detailed balance, MCMC	Chapter 3
Jan 29 - Feb 2	Continuous-time Markov chains	Chapter 4
February 5-9	Gaussian processes & stationary processes	Chapter 5

February 12-16	Brownian motion	Chapter 6
February 19-23	Spring break	
February 26 - March 1	Stochastic integration	Chapter 7
March 4-8	Stochastic differential equations	Chapter 8
March 11-15	Numerically solving SDEs	Chapter 9
March 18-22	Forward & Backward equations for SDEs	Chapter 10
March 25- March 28	Some applications of the backward equation	Chapter 11
April 2-5	Detailed balance, symmetry, eigenfunction expansions	Chapter 12
April 8-12	TBD	TBD

Policies

Weather Contingency Plan

You should check ubc.ca often during bad weather or snow. If a class session is cancelled, class will be held online. Instructions will be posted on Canvas. For those unable to participate in the online class, we will provide a recording on Canvas. If a cancellation impacts a midterm, we will reschedule to another class time.

University policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the UBC Senate website.

Course Summary:

Date	Details	Due
Thu Jan 18, 2024	**Homework 1** (https://canvas.ubc.ca/courses	due by 11:59pm

Date	Details	Due
	/139375/assignments/1770481)	