Math 428/513: 539: Analytic Number Theory
Spring Term, 2027

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Course Website	`http://www.math.ubc.ca/~lior/teaching/2627/539_W27/`
Contact me at	`lior@math.ubc.ca` or at MATX 1112
My Website	`http://www.math.ubc.ca/~lior/`
Class	TTh 9:30-11:00 at ESB 4127 and on Zoom (link on course website)
Office Hours	Fridays 10:30-11:30 and on Zoom.
Piazza board	TBA
Textbook	None required; see below for recommendations
(Informal) Prerequisites	Elementary Number Theory, Real Analysis, Complex Analysis

About the course

e will count (that is, estimate the number of) integer and prime number solutions to equations. We will use combinatorial ("elementary") methods, some Fourier analysis, and finally zeta-function (contour integration) techniques. Possible topics include:

- Elementary techniques: Divisor sums; the Chebychev and Mertens estimates.
- Fourier analysis and exponential sums. Smooth counting.
- Dirichlet series and the Mellin transform; countour shifting.
- The Riemann zeta function; analytical continuation; the Prime Number Theorem.
- Dirichlet L-functions and the Prime Number Theorem in Arithmetic Progressions.

The main pre-requisites are Elementary Number Theory and Real and Complex analysis (say at the level of UBC MATH 537, 320, and 508, respectively). We will use some basic ideas from ring theory and finite abelian groups, and will develop all the Fourier analysis we will use.

Textbooks

There are many good textbooks in the field. Davenport [1] covers the material tersely. Montgomery–Vaughn [3] have more details and exercizes. The tome by Iwaniec–Kowalski [2] contains a lot more material, and usually works at a great level of generality.

Evaluation and grading; levels of participation

On the use of external resources

By necessity many homework problems in this course are *standard*, or involve proofs of standard results which may be found in many websites and textbooks. Please submit only your own work and refrain from submitting solutions you found online, in textbooks, or obtained from experts. It is normal to talk to each other and to experts (or GenAI) as you *learn* the material, but you may not submit answers substantially created by others. Please cite specifically any external resource or person you have used. In particular, the use of generative AI tools to complete or support the completion of any assignment in this course is not allowed and would be considered academic misconduct.

Graduate students

Graduate students may take the course for different reasons — ranging from planning to conduct research in analytic number theory to pure personal interest — and have widely differing background. They should accordingly calibrate their efforts depending on their background and personal learning goals.

- Students should communicate to the instructor early in the term their goals in the course. Students should do as much work on each problem set as they deem appropriate.
- Graduate students will be judged and graded appropriate to their goals.

Remote participation

The course will be simultaneously broadcast on Zoom and open for remote participation, including as a PIMS network-wide course. The Zoom link is posted to the course website.

Registration

I would appreciate remote students interested in the course to send me an email describing their physics and mathematics experience and their intended level of participation (see above).

1. Students at Canadian PIMS member universities may apply for graduate credit via the Western Deans' Agreement. Please be advised that in some cases students must enroll 6 weeks in advance of the term start date and will typically be required to pay ancillary fees to the host institution (sometimes as much as $270) or to explicitly request exemptions from such fees. Please follow the link above for details of fees at specific sites.
2. Students at universities not covered by the WDA but which are part of the Canadian Association for Graduate Studies may still be eligible to register for this course under the Canadian University Graduate Transfer Agreement. Details of this program vary by university and are also typically subject to ancillary fees. Please contact your local Graduate Student Advisor for more information.
3. As an alternative to formal registeration remote students (from any university) may enrol in a local "reading course" at their home university. In that case please have the faculty member responsible for the reading course (usually your advisor) write to me to request that I grade your homework and send a grade back.
4. Anyone (anywhere in the world) who is interested in mathematics and would like to learn some mechanics is welcome to attend the lectures as well as the online office hours. I will only mark the homework of (or otherwise perform "official duties" for) registered participants.

Online classes

1. All remote and in-person participants are strongly encourage to **interrupt the lectures** repeatedly with their questions (even "can you explain that again?" or "what do you mean by 'continuous'?"). Such questions are welcome, encouraged, and are **absolutely essential** for learning to take place in a course of this type.

2. All classes will be recorded, with the recordings placed on a public website (likely YouTube). Participants who do not wish to be recorded may join the Zoom session under pseudonyms, and only ask questions via chat.

3. There will be online office hours to support the remote participants.

4. All course information will be posted on the course website and will remain accessible in perpetuity to anyone anywhere.

Boilerplate

1. The mathematics department's General Syllabus Information applies to this course.

2. In case of inclement weather the course may revert to Zoom-only.

References

[1] Harold Davenport. *Multiplicative number theory*, volume 74 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, third edition, 2000. Revised and with a preface by Hugh L. Montgomery.

[2] Henryk Iwaniec and Emmanuel Kowalski. *Analytic number theory*, volume 53 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 2004.

[3] Hugh L. Montgomery and Robert C. Vaughan. *Multiplicative number theory. I. Classical theory*, volume 97 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2007.