University of Alberta
Department of Mathematical & Statistical Sciences

PIMS network Course on Stochastic Differential Equation under the University of Alberta Course Stochastic Analysis II MATH 510 Winter 2027 (January - April 2027)

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Office Hours:	To be determined and/or by appointment via zoom
Lecture Room:	Online Via Zoom
Lecture Time:	Tuesdays, Thursdays 11:00am - 12:20pm

Course Description:

This is a three credit course topic course at University of Alberta. It should be under an existing topic course number: MATH 510 Stochastic Analysis II with a subtitle: Stochastic differential equations. Students from other universities can enrol this course to get credits via Western Deans' agreement.
It is lectured and recorded via zoom.

Abstract:

This is a one semester three credit hour course.
It is about the theory and applications of stochastic differential equations driven by Brownian motion.
A stochastic differential equation (SDE) is a differential equation in which the rate of change is determined by the state of the system itself, some external known forces and some unknown external forces as well. The noise in the system is given by random white noise calculated as the derivative of Brownian

motion or the Wiener process. However, other types of random behaviour are possible, such as jump processes. Random differential equations are conjugate to stochastic differential equations. This course will concentrate on stochastic differential equations driven by Brownian motions.
The stochastic differential equations are used to model various phenomena such as unstable stock prices or physical systems subject to thermal fluctuations. They have found applications in finance, signal processing, population dynamics and many other fields. It is the basis of some other applied probability areas such as filtering theory, stochastic control and stochastic differential games. To balance the theoretical and applied aspects and to include as much audience as possible, we shall focus on the stochastic differential equations driven only by Brownian motion (white noise). We will focus on the theory and not get into specific applied area (finance, signal processing, filtering, control and so on).
We shall first briefly introduce some basic concepts and results on stochastic processes, in particular the Brownian motions. Then we will discuss stochastic integrals, Itô formula, the existence and uniqueness of stochastic differential equations, some fundamental properties of the solution. We will concern with the Markov property, Kolmogorov backward and forward equations, Feynman-Kac formula, Girsanov formula. We will also concern with the ergodic theory and other stability problems. We may also mention some results on numerical simulations, Malliavin calculus and so on.

Course Prerequisites:

Some preparation on mathematical analysis and probability theory based on measure theory such as STAT 571 or Stat 580 is not necessarily required but will be very much helpful.

Course Objectives and Expected Learning Outcomes:

Students are expected to understand the basic material of stochastic differential equation and know what is its goals and what is the mathematical tools it uses. Stochastic differential equations can be a very theoretical mathematical branch. But it also has many immediate applications to various fields. Students are expected to have a solid background and analytic skills. This will also be necessary for their future study of some other fields such as statistics, engineering, mathematical finance, machine learning.

The course contents are chosen so that they may be applicable to wide range of areas so researchers from various fields may be interested in these materials. This course is not specifically designated to students in finance or students in electrical engineering. But to a wider audience who want to have some more knowledge on the modern state-of-art theory of stochastic differential equations. The course will be delivered through remote means. There will be slides for each lecture. We will use the writing on the screen to discuss problems and answers so that it is as close as to the classroom presentation.

Required Learning Resources:

The main reference books for this course are

- Øksendal, B. Stochastic differential equations. An introduction with applications. Sixth edition. Universitext. Springer-Verlag, Berlin, 2003. xxiv+360 pp. ISBN: 3-540-04758-1
- Karatzas, Ioannis; Shreve, Steven E. Brownian motion and stochastic calculus. Second edition. Graduate Texts in Mathematics, 113. Springer-Verlag, New York, 1991. xxiv+470 pp. ISBN: 0-387-97655-8
- Klebaner, Fima C. Introduction to stochastic calculus with applications. Third edition. Imperial College Press, London, 2012. xiv+438 pp. ISBN: 978-1-84816-832-9; 1-84816-832-2

Recommended or Optional Learning Resources:

There are many other nice books on general basic stochastic processes. Students are encouraged but not required to read one or some of them.

Some other reference books are

- Ikeda, N.; Watanabe, S. Stochastic differential equations and diffusion processes. Second edition. North-Holland Mathematical Library, 24. North-Holland Publishing Co., Amsterdam; Kodansha, Ltd., Tokyo, 1989. xvi+555 pp. ISBN: 0-444-87378-3
- Protter, P. E. Stochastic integration and differential equations. Second edition. Version 2.1. Corrected third printing. Stochastic Modelling and Applied Probability, 21. Springer-Verlag, Berlin, 2005. xiv+419 pp. ISBN: 3-540-00313-4
- Revuz, D.; Yor, M. Continuous martingales and Brownian motion. Third edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 293. Springer-Verlag, Berlin, 1999. xiv+602 pp.
- Durrett, R. Stochastic calculus. A practical introduction. Probability and Stochastics Series. CRC Press, Boca Raton, FL, 1996. x+341 pp. ISBN: 0-8493-8071-5
- Jeanblanc, M.; Yor, M.; Chesney, M. Mathematical methods for financial markets. Springer Finance. Springer-Verlag London, Ltd., London, 2009. xxvi+732 pp. ISBN: 978-1-85233-376-8
- Hasminskii, R. Z. Stochastic stability of differential equations. Translated from the Russian by D. Louvish. Monographs and Textbooks on Mechanics of Solids and Fluids: Mechanics and Analysis, 7. Sijthoff &

Noordhoff, Alphen aan den Rijn—Germantown, Md., 1980. xvi+344 pp. ISBN: 90-286-0100-7

- Hu, Y. Analysis on Gaussian spaces. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017. xi+470 pp. ISBN: 978-981-3142-17-6
- Kloeden, P. E.; Platen, E. Numerical solution of stochastic differential equations. Applications of Mathematics (New York), 23. Springer-Verlag, Berlin, 1992. xxxvi+632 pp. ISBN: 3-540-54062-8

Grade Evaluation:

The course mark will be calculated based on the following breakdown:

Course Component	**Weight**	**Date**
Assignments	50%	Due every Tuesday unless otherwise announced
Midterm	20%	To be determined some day in February
Final Exam	30%	To be determined

*Note: The date of the final examination is set by the Registrar and takes precedence over the final examination date reported in this document. Students must verify this date on BearTracks when the Final Exam Schedule is posted. The final letter grade will be determined from the course mark as follows: Grades are unofficial until approved by the Department and/or Faculty offering the course.

Assignments:

(i) Homework will be assigned after each lecture.

(ii) Homework is submitted electronically through "Assign2". You can access to the eclass through https://eclass.srv.ualberta.ca using your ccid.

(iii) The electronic submission of homework is due on Monday at 23:00pm for the two assignments of the previous week unless otherwise stated.

Exam Format:
The format (not the content) of all midterm exams and final will look like

TOTAL: _____

Student Name : _____

Exam for Math 510, Stochastic Analysis II/Stochastic Differential Equations

Problem 1 A bowl contains twenty cherries, exactly fifteen of which have had their stones removed. A greedy pig eats five whole cherries, picked at random, without remarking on the presence or absence of stones. Subsequently, a cherry is picked randomly from the remaining fifteen.

(a) What is the probability that this cherry contains a stone?

(b) Given that this cherry contains a stone, what is the probability that the pig consumed at least one stone?

Problem 2 A purse contains 12 quarters and 2 pennies. All the coins are to be drawn, one at a time, without replacement. You keep all the quarters that are drawn between the two pennies, which is denoted by X (The total number of quarters between the two pennies). Find $\mathbb{E}(X)$ and $\text{var}(X)$.

Exam Aids:

All exams are close book exam. This means it is not allowed to use textbook or any other books. However, Students are allowed to use all kinds of calculators

(but no laptop or cellphone can be used). They are also allowed to use one normal size (8×11) page of note which can be written on both sides.

Excused Absence Where the Cause is Religious Belief:

For an excused absence where the cause is religious belief, a student must contact the instructor(s) within two weeks of the start of Fall or Winter classes to request accommodation for the term (including the final exam, where relevant). Instructors may request adequate documentation to substantiate the student request.

Missed Term Work

A student who cannot write a midterm To apply for an excused absence, a student must inform the instructor within two working days following the scheduled date of the term work or term exam missed, or as soon as the student is able, having regard to the circumstances underlying the absence. In all cases, instructors may request adequate documentation to substantiate the reason for the absence at their discretion.
An excused absence is a privilege and not a right; there is no guarantee that an absence will be excused. Misrepresentation of Facts to gain an excused absence is a serious breach of the *Code of Student Behaviour*.

Missed Final Examination:

A student who cannot write the final examination due to incapacitating illness, severe domestic affliction or other compelling reasons can apply for a deferred final examination. Students who failed at the start of term to request exam accommodations for religious beliefs are expected to follow the normal deferred final examination process. Such an application must be made to the student's Faculty office within two working days of the missed examination and must be supported by a Statutory Declaration (in lieu of a medical statement form) or other appropriate documentation (Calendar section 23.5.6). Deferred examinations are a privilege and not a right; there is no guarantee that a deferred examination will be granted. Misrepresentation of Facts to gain a deferred examination is a serious breach of the *Code of Student Behaviour*.

Any deferred final examinations are scheduled as follows:
All approved Deferred Final Exams for the upcoming Winter Term 2027 will be held on at some specific time to be determined (TBA) later.

Date: TBA
Time: TBA
Location: TBA

Re-examination:

A student who writes the final examination and fails the course may apply for a re-examination. Re-examinations are rarely granted in the Faculty of Science. These exams are governed by University (Calendar section 23.5.5) and Faculty of Science Regulations (Calendar section 192.5.3). Misrepresentation of Facts to gain a re-examination is a serious breach of the *Code of Student Behaviour*.

STUDENT RESPONSIBILITIES

Academic Integrity:

The University of Alberta is committed to the highest standards of academic integrity and honesty. Students are expected to be familiar with these standards regarding academic honesty and to uphold the policies of the University in this respect. Students are particularly urged to familiarize themselves with the provisions of the *Code of Student Behaviour* (online at www.governance.ualberta.ca) and avoid any behaviour which could potentially result in suspicions of cheating, plagiarism, misrepresentation of facts and/or participation in an offence. Academic dishonesty is a serious offence and can result in suspension or expulsion from the University.
All forms of dishonesty are unacceptable at the University. Any offense will be reported to the Senior Associate Dean of Science who will determine the disciplinary action to be taken. Cheating, plagiarism and misrepresentation of facts are serious offenses. Anyone who engages in these practices will receive at minimum a grade of zero for the exam or paper in question and no opportunity will be given to replace the grade or redistribute the weights. As well, in the Faculty of Science the sanction for **cheating** on any examination will include **a disciplinary failing grade** (NO EXCEPTIONS) and senior students should expect a period of suspension or expulsion from the University of Alberta.

Collaboration on Assignments:

Students should work alone for each of their assignments. But they can collaborate to work on the homework assignments. However, the midterm exams are not collaborative and close book.

Exams:

Students will not be allowed to begin an examination after it has been in progress for 30 minutes. Students must remain in the exam room until at least 30 minutes has elapsed. Electronic equipment cannot be brought into examination rooms.

Cell Phones:

Cell phones are to be turned off during lectures, labs and seminars. Cell phones are not to be brought to exams.

Audio or Video Recording:

Audio or video recording, digital or otherwise, of lectures, labs, seminars or any other teaching environment by students is allowed only with the prior written consent of the instructor or as a part of an approved accommodation plan. Student or instructor content, digital or otherwise, created and/or used within the context of the course is to be used solely for personal study, and is not to be used or distributed for any other purpose without prior written consent from the content author(s).

Students Eligible for Accessibility-Related Accommodations (students registered with Student Accessibility Services – SAS):

Eligible students have both rights and responsibilities with regard to accessibility-related accommodations. Consequently, scheduling exam accommodations in accordance with SAS deadlines and procedures is essential. Please note adherence to procedures and deadlines is required for U of A to provide accommodations. Contact SAS (www.ssds.ualberta.ca) for further information.

Student Success Centre:

Students who require additional help in developing strategies for better time management, study skills, or examination skills should contact the Student Success Centre (2-300 Students' Union Building).

Decima Robinson Support Centre for Mathematical & Statistical Sciences:

Students who require additional help with assignments or have questions about the course material in general are encouraged to visit the Decima Robinson Support Centre (528 Central Academic Building). Graduate students will be available to provide one-on-one help. In order to get maximum help during each visit, students are asked to be specific about the problem with which they are seeking help. The Centre is open Monday to Friday, 9:00–15:00.

Policy about course outlines can be found in section 23.4(2) of the University Calendar.

Disclaimer:

Any typographical errors in this Course Outline are subject to change and will be announced in class.

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