An advanced graduate topics course: An Introduction to Symbolic Dynamics:

Math 601D, Topics in Analysis, Sec 201, UBC-V, January-April, 2027, hybrid: in-person at UBC + PIMS online course + others (with instructor's permission), MWF 1-2, CHEM C124.

Instructor: Brian Marcus, Professor of Mathematics, UBC and PIMS/UBC Site Director

Delivery method: Ipad

References:

- [B] Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms, R. Bowen (with revisions by Jean-Rene Chazottes), 2nd edition, 2017, Springer Lecture Notes in Mathematics, 470, Springer
- [LM] An Introduction to Symbolic Dynamics and Coding, D. Lind and B. Marcus, 2nd edition, 2021, Cambridge Mathematical Library, Cambridge University Press
- [W] An Introduction to Ergodic Theory, P. Walters, 1982, Springer Graduate Texts in Mathematics, 79, Springer

([B] and [LM] are available online in the UBC library, as are several introductory ergodic theory textbooks)

Brief description:

Symbolic Dynamics is a part of the theory of dynamical systems, with strong connections to ergodic theory and information theory. This course will serve as an introduction to symbolic dynamics, including shifts of finite type, sofic shifts, entropy, Gibbs meauures, equilibrium meaasures, and problems of encoding and decoding from one symbolic system to another. Emphasis on Z-symbolic dynamics (i.e. spaces of one-dimensional sequences), with some material on G-symbolic dynamics (where G is a countable group). There is a very rich history of major results in this area, and many open problems.

Prospective audience: current PhD and MSc students interested in ergodic theory, dynamical systems, information theory, probability theory, and fractal geometry.

Prerequisites:

Measure Theory (420/507), Functional Analysis (421/510), Probability 1 (418/544), Probability 2 (419/545), strong undergraduate courses in linear and abstract algebra, some previous exposure to the basics of ergodic theory will be helpful (this will be reviewed in class).

Course Topics:

1. Shift spaces [LM, Ch 1 and Sec. 6.2, 6.3]
2. Shifts of Finite Type (SFTs) [LM, Ch 2]
3. Sofic shifts [LM, Ch 3]
4. Minimal shifts: Morse shift and Sturmian shifts [LM, Sec 13.7]

5. Classical Perron Frobenius Theory [LM, Ch 4]
6. Topological entropy [LM, Ch 4]
7. A quick introduction to ergodic theory [W, Ch. 1,2,4]
8. Measures of maximal entropy for SFTs [LM, Sec. 13.3]
9. Ruelle Perron Frobenius Theory [B, Sec. 1.A,B]
10. Gibbs measures and equilibrium states [B, Sec 1.C,D]
11. Classification of SFTs up to topological conjugacy [LM, Ch. 7]
12. Other coding problems for SFTs and sofic shifts in symbolic dynamics (e.g., almost topological conjugacy, embeddings, factor maps) [LM, parts of Ch. 5, 8, 9, 10, 12]
13. Markov partitions [LM. Sec. 6.5]
14. Symbolic dynamics for G-shifts, where G is a countable group [LM, sec. A.8]

Course requirements: There will be three homework sets and each student is expected to give one course lecture.

For questions, please contact the instructor, Brian Marcus:

marcus@math.ubc.ca

(604) 328-1471 (cell phone)

https://personal.math.ubc.ca/∼marcus/